ARGUMENTA
OECONOMICA
CRACOVIENSIA

No 15 • 2016
ISSN 1642-168X
AOC, 2016; 15: 91–107
DOI: 10.15678/AOC.2016.1506

Grażyna Trzpiot
Justyna Majewska

MODELLING LONGEVITY RISK IN THE CONTEXT OF CENTRAL STATISTICAL OFFICE POPULATION PROJECTIONS FOR POLAND TO 2050

Abstract

The problem of an ageing population confronts the majority of advanced countries. This paper analyses the probability, which may be termed the probability of a sustainable pension, that a retired person will not face financial ruin before they die.

Keywords: longevity risk, life expectancy projections, contribution pension plan, propability of ruin.
JEL Classification: C130, C180.

1. Introduction

Demographic ageing is the result of people living longer as mortality rates fall. In the majority of countries, the length of time people are expected to live has increased by 25–30 years during the last century. Of the social, political, economic and regulatory challenges presented by constant improvements in longevity, the consequences for pensions have perhaps received the most publicity (Barrieu et al. 2012). If improvements in life expectancy could be predicted, and taken into account when planning retirement, they would have a negligible effect on retirement finances

Grażyna Trzpiot, University of Economics in Katowice, Department of Demography and Economic Statistics, Bogucicka 14, 40-226 Katowice, Poland, e-mail: grazyna.trzpiot@ue.katowice.pl

Justyna Majewska, University of Economics in Katowice, Department of Demography and Economic Statistics, Bogucicka 14, 40-226 Katowice, Poland, e-mail: justyna.majewska@ue.katowice.pl

(Antolin 2007). Unfortunately, gains in mortality and life expectancy are uncertain. In this regard, longevity risk is associated with the risk that future mortality and life expectancy will not be as expected (Antolin 2007).

Rising life expectancy increases the risk that people will outlive the financial resources they have set-aside for retirement. For an insurer or a pension scheme, improving mortality rates raises the risk that pay-outs will exceed forecasts. There are roughly two types of longevity risk. The first, non-systematic risk, arises from random fluctuations between individuals and can be mitigated by increasing the size of portfolios, while the second, systematic risk, affects all individuals in a non-random manner and cannot be diversified by pooling. People are likely to be more concerned about non-systematic risks, while insurers are likely to be more concerned about managing systematic risks.

The main purpose of this paper is to understand how uncertainty regarding life-expectancy outcomes affect the liabilities of defined-contribution private pension plans provided by employers. To do so, the paper first focuses on assessing the uncertainty surrounding future developments in life expectancy, that is, longevity risk. Secondly, it examines the impact that longevity risk could have on defined-contribution pension plans provided by employers. In this paper we investigate the effect of systematic longevity risk.

2. Global Demographic Change

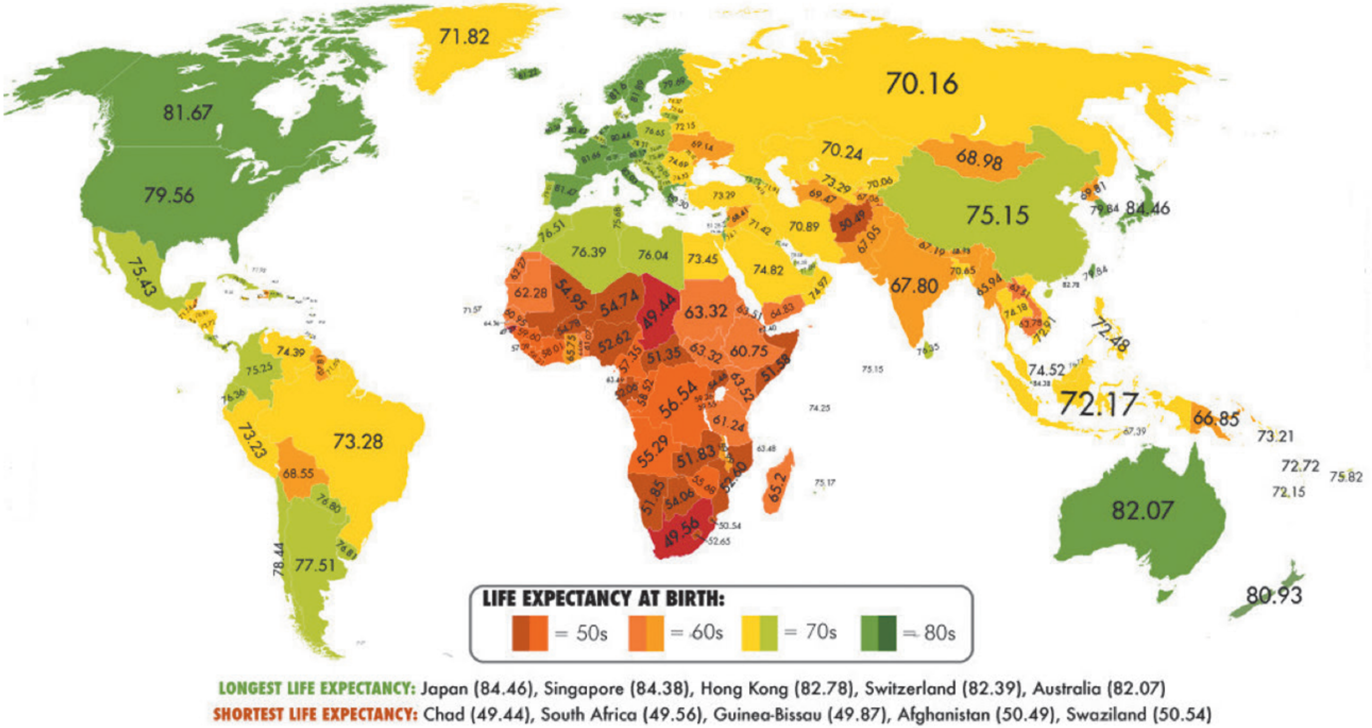
Increased life expectancy is a worldwide phenomenon. Improvements in health and the related rise in life expectancy are among the most remarkable demographic changes of the past century.

There are two ways in which the population may age (Arltová, Langhamrová & Langhamrová 2013):

– relative ageing of the population caused by a fall in the birth rate and the consequent fall in the number of children in the population,

– absolute ageing caused by a fall in mortality; there are then greater numbers of older people in the population due to rising life expectancy.

Whether the population becomes younger or older depends on the nature of the age structure in the past, and on current birth rate and mortality. High mortality rates in the past meant that life expectancy at birth was shorter. Global life expectancy, which rose from approximately 30 years in 1900 to 65 years in 2000, more than doubled in the twentieth century; it is forecast that it will have risen to 81 by the end of the twenty-first. Over the second



LONGEST LIFE EXPECTANCY: Japan (84.46), Singapore (84.38), Hong Kong (82.78), Switzerland (82.39), Australia (82.07)
SHORTEST LIFE EXPECTANCY: Chad (49.44), South Africa (49.56), Guinea-Bissau (49.87), Afghanistan (50.49), Swaziland (50.54)

Fig. 1. Global Life Expectancy at Birth in 2013. Country Perspective

Source: CIA (2014).

half of the twentieth century, global life expectancy at birth increased by four-and-a-half months per year (2011), which amounts to a change of more than 18 years. The same upward trend is occurring in North America, South America, Europe, and Asia (Figure 1, Figure 2).

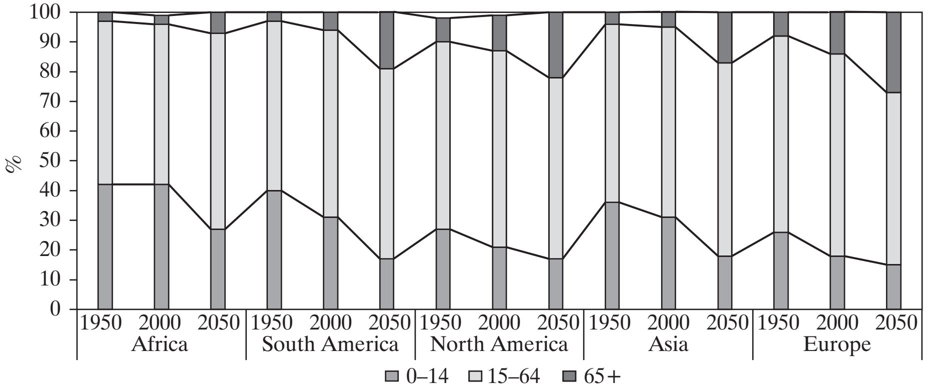


Fig. 2. Historical Trends and Projection of Age Group Shares in Selected Continent Populations

Source: www.un.org/en. Accessed: 10 March 2015.

Table 1. Longevity Trends (in Years), 1970–2050

Countries and regions	Observed			Projected	
	1970–2010	Increase per year	Standard deviation	2010–2050	Increase per year
Change in life expectancy at birth					
USA and Canada	8.2	0.20	0.14	4.3	0.11
Advanced Europe	8.6	0.21	0.13	4.7	0.12
Emerging Europe	1.1	0.03	0.36	6.8	0.17
Australia and New Zealand	10.8	0.27	0.27	4.9	0.12
Japan	10.8	0.27	0.23	4.6	0.11
Change in life expectancy at 60					
USA and Canada	4.9	0.12	0.11	3.1	0.08
Advanced Europe	5.7	0.14	0.13	3.7	0.09
Emerging Europe	0.6	0.02	0.18	3.8	0.09
Australia and New Zealand	7.2	0.18	0.23	3.7	0.09
Japan	7.7	0.19	0.19	3.7	0.09

Source: Human Mortality Database (13 December 2011) and IMF staff estimates.

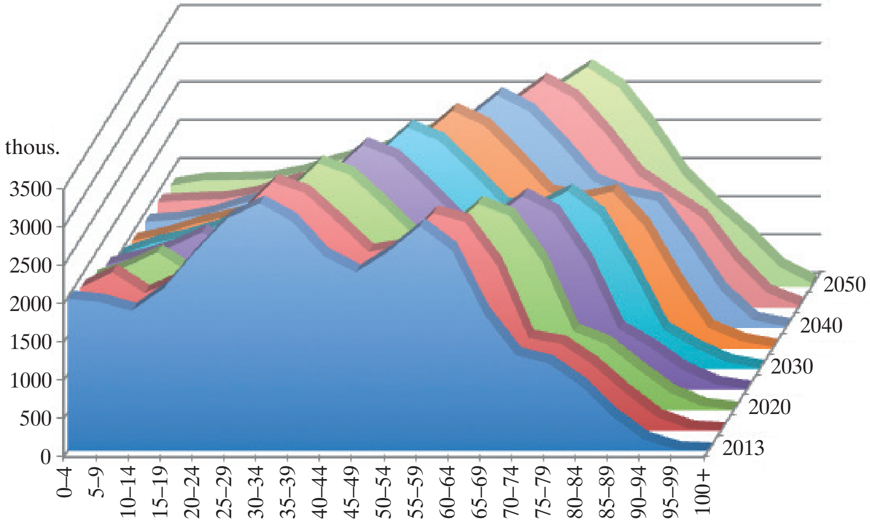


Fig. 3. Poland: Population by Age and Year

Source: population projection according to CSO in Poland, www.stat.gov.pl. Accessed: 10 March 2015.

The main source of longevity risk is the disparity between expected lifespans and actual lifespans, which has at times been considerable. Regardless of the technique used, forecasters have tended to consistently underestimate how long people will live (IMF 2012).

In 2009, numerous companies in developed economies closed their defined-benefit retirement plans. This represented a transfer of risk from industry and insurers back to policyholders. From a social point of view, this is no longer regarded as satisfactory. A number of countries, however, have been replacing defined-benefit pension plans with defined-contribution plans. But this has only resulted in the same unsatisfactory transfer of risk. Prompted by longevity improvements, ageing populations and the need to raise more finance for pensions, a number of governments are now planning to add an additional two to five years to the retirement age.

As Figure 3 illustrates, the changes that will occur in demographic age profiles will not leave Poland untouched. The average proportion of the population aged 60+ throughout our sample is projected to have increased to 29% in 2030 (compared to 16% in 1970), with most of the corresponding decline sustained by the group aged 0–19.

3. The Link between Mortality and Life Expectancy: Life Tables

In providing a summary description of mortality, survivorship, and life expectancy for a specified population, life tables represent a link between mortality and life expectancy. Complete life tables contain data for every single year of age, while abridged life tables contain data for five-year intervals and ten-year intervals. In its simplest form, a life table can be generated from a set of age-specific death rates (ASDR) which, based on vital statistics, are calculated as the ratio of the number of deaths during a year to the corresponding population size, which in turn is derived from censuses and annual estimates.

The final outcome of a life table is the mean number of years still to be lived by a person who has reached a specific age (hence age-specific life expectancies), if the current age-specific probabilities of dying are applied for the rest of their life.

In detail, this means that for each $x \in N$ up to a maximum age of, say, 120 (ignoring for the sake of clarity both truncated observations and cases of censored data, in which an individual's time of death is not precisely known), we consider the number l_x of individuals who turn age x. Assuming that d_x out of those l_x individuals will die between age x and $x + 1$, the annual mortality rate q_x at age x is the probability that someone aged x will die within one year. This can be estimated by l_x/d_x.

4. Longevity Risk

According to the NAIC definition (2010), this is the risk that actual survival rates and life expectancy will exceed expectations or pricing assumptions, resulting in a need for greater-than-anticipated cash flows for retirement. For individuals, this is the risk of outliving one's assets, which can lead to a lower standard of living, reduced care or a return to employment. For institutions that provide a guaranteed retirement income to people who are covered, longevity risk means underestimating survival rates. This results in increased liabilities and insufficient funds to make promised payments (NAIC 2010). The key drivers of the growing need to address longevity risk include an ageing population, increasing life expectancy, a shift in the locus of responsibility for providing a sufficient retirement income, the uncertainty of government benefits and economic volatility (NAIC 2010).

There are numerous holders of longevity risk. Principally they are governments, but they are also employers, individuals and insurers. There are various ways in which risk can be passed from one of these parties to another (Figure 4).

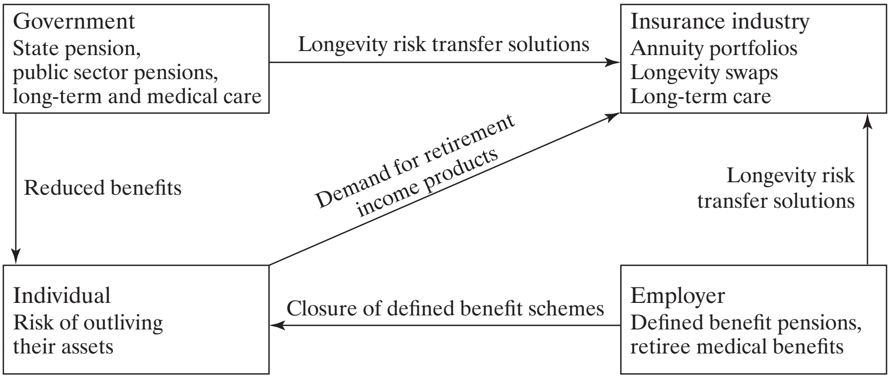


Fig. 4. The Holders of Longevity Risk

Source: Osorio (2013, p. 27).

There now follows an account of the description given by Swiss Re in 2014 of the relationships between holders of longevity risk. Given they undertake to pay retirees an income via a state pension, provide defined-benefit pensions for state employees and meet healthcare commitments, governments are influenced by an ageing society in many ways, all of which create significant liabilities. In an attempt to tackle this menace, many of them are beginning to reduce benefits in real terms, so that the burden placed on the individual to provide an income in retirement grows heavier. Employers who sponsor their employees' retirement incomes via defined-benefit plans, and employers who offer medical benefits to retired employees, will be concerned about the impact longevity can have on their future liabilities. To ameliorate this situation, many employers have closed plans down and replaced them with defined-contribution pensions, which has increased the risk burden on the individual still further. Given the declining amounts states and employers provide for retirement income, the responsibility placed on the individual is growing sharply. People are now expected to establish defined-contribution plans for their retirement and to address the risks associated with inflation, assets and longevity. There are therefore concerns that people will outlive the assets they have accumulated, which leaves a gap that the state is increasingly unable to fill. People are thus faced with the severe challenge of preparing for a stage in their lives when expenses, such as for long-term care, can be expected to increase. One solution is to work longer. But this depends on employment being available and on people being fit enough to do it. A well-diversified insurer will combine mortality risk, which is the risk that people will die sooner than expected, with longevity risk and other non-correlated insurance perils, such as property and casualty. It is this type of diversification, balancing two opposing risks, and diversifying across a portfolio of insurance perils, that in many cases makes insurers the natural home for longevity risk. (Re)insurers offer a range of solutions that can help governments, employers and individuals to pass on some or all of their longevity risk.

Like systematic risk, longevity risk is not diminished by diversification. In short, longevity risk is real, global, and non-diversifiable.

5. Modelling and Projecting Longevity

The close relationship between mortality and longevity modelling appears clear when we consider survival probability. Mathematically, life expectancy would appear to be the product of correlated mortality rates, which is supported by the following expression for the survival probability until date $t + u$ of a person aged x at time t (Barrieu et al. 2012):

$$S_t(x,T) = \prod_{i=0}^{T-1} [1 - q(x+i, t+i)]. \tag{1}$$

Mortality models are usually used for both mortality and longevity risks.

The literature contains several approaches to the projection of mortality rates (Wong-Fupuy & Haberman 2004). Public pension systems, or private pension funds, providing defined pension benefits, require mortality projections to determine the number of people who will be entitled to a pension.

The three main ways of modelling life expectancy are (1) a method based on underlying biomedical processes, (2) methods based on explanation that employ causal forecasting and econometric relationships and (3) methods of extrapolation that take historical mortality trends and project them forward. It is worth noting that these approaches are usually combined.

Models based on extrapolation are the ones that actuaries, official organizations and national statistical offices use most often. They employ past data to express age-specific mortality as a function of calendar time and, as such, can be deterministic or stochastic (Antolin 2007). The main difference between these models is that deterministic models do not take uncertainty of life expectancy into account, which means that they are not equipped with standard errors or projection probabilities. The literature distinguishes extrapolative stochastic methods that are based on (1) the interdependent projection of age-specific mortality (including graduation models, CMI), (2) standard time series procedures such as the Lee-Carter method (Lee & Carter 1992), where a log-linear trend for age-specific mortality rates is often assumed for the time-dependent component and (3) econometric modelling, of which *P*-spline models offer an example (Antolin 2007).

National statistical offices tend, however, to extrapolate historical trends in a deterministic way, while actuaries use stochastic approaches that are more sophisticated. What is more, national statistical offices and actuaries use different populations for their mortality and life expectancy projections. From the mortality tables they produce, national statistical offices project life expectancy for the entire populations of their countries. But the mortality rates of participants in private pension plans can differ substantially from those of the overall population, which is why these plans use their own actuarial mortality tables. It is a well-known fact that mortality rates are lower, and life expectancy is higher, for women and for well-educated, high-income individuals (Goldman 2001, Drever, Whitehead & Roden 1996). The use of life tables differentiated by socio-economic group can, however, give rise to a different set of problems (Antolin 2007).

Table 2. Projected Life Expectancy in Poland until 2050

Year	A1		A2		A3	
	men	women	men	women	men	women
2013 (real data)	73.1	81.1	73.1	81.1	73.1	81.1
2015	73.5	81.5	73.6	81.5	73.7	81.6
2020	74.9	82.5	75.0	82.6	75.3	82.7
2025	76.3	83.6	76.6	83.8	77.2	84.0
2030	78.0	84.8	77.5	84.4	78.3	84.8
2035	39.1	85.6	78.5	85.2	79.6	85.7
2040	80.3	86.5	79.5	85.9	80.9	86.7
2045	81.6	87.4	80.6	86.7	82.4	87.8
2050	83.0	88.4	81.8	87.6	84.1	88.9

Source: *Population Projection 2014–2015* (2014).

In Poland, the majority of population forecasts (and therefore of life expectancy forecasts) are based on deterministic models and are calculated in low, medium and high variants of future development:

– medium variant (A1) – the “delay” of Polish mortality in relation to the developed countries will be maintained at the same level throughout the forecast period,

– low variant (A2) – the “delay” of Polish mortality will remain at the same level until 2025; thereafter the pace of reduction in mortality will slow down,

– high variant (A3) – the distance between Poland and the developed countries will gradually decline throughout the forecast period.

In each variant the demographic factors are estimated based on the extrapolation of actual values and include a number of preconditions for the development of the individual components of population development.

6. Capital Requirements and the Probability of Ruin

Future mortality and life expectancy should be estimated using a stochastic approach which, by attaching probabilities to different outcomes, makes it possible to assess uncertainty and risk. Future developments in mortality rates and life expectancy are uncertain, but some paths or trajectories are more likely than others (*Pension Fund…* 2010). Forecasts of mortality and life expectancy should therefore consider a range of the most likely outcomes and take account of the related probabilities. There is a trade-off between greater certainty and greater precision.

If a pension system is based on the fund principle we must decide how much to “save” annually during the accumulation phase and how much to “spend” annually during the decumulation (annuity) phase (Cipra 2010). In view of the many random aspects, the best approach is similar to that applied in modern finance: Value at Risk, whereby the highest loss that can occur with a given probability (tolerance) is calculated. In the context of pensions, this must be modified to the probability that the retired person will not be “ruined” before the moment of death (the probability of a sustainable pension). Its obverse is the probability of ruin (the probability of an unsustainable pension). This is closely connected with the practise of pension planning or of managing the risk of pensions (Cipra 2010). In terms of internal models, the Solvency II guidelines propose using Value at Risk to compute the capital required when an insurer prefers to develop its own framework for risk assessment (Barrieu et al. 2012). The methodology considered here is very different from that now in use in the banking industry[1].

Where defined-contribution plans are concerned, contributions are in most cases defined in advance as a percentage of a participant's salary. The pension should be sufficient to provide an adequate income for the rest of a participant's life, and possibly also that of a partner, and should remove the risk that participants will outlive their resources.

At the age of retirement, for example sixty-five, capital of w is accumulated in the participant's account, which will be decumulated by the corresponding annual pension payments (Cipra 2010). The pension plan is stochastic and supposes that benefits follow a geometric Brownian motion. In modern finance, the randomness of interest rates on the capital invested from the participant's account is usually modelled by geometric Brownian motion (Malliaris & Brock 1982). Here, capital S_t in time t can be evaluated beginning with capital S_0 (in time 0) as (Cipra 2010):

$$S_t = S_0 \cdot e^{B_t(\mu,\sigma)} = S_0 \cdot e^{\mu t + B_t \sigma}, \tag{2}$$

where B_t is the classical Brownian process, μ is the drift modelling the trend of the capital investment, and σ is the volatility modelling the diffusion of the capital investment. Note that S_t has a log-normal distribution.

The second aspect of the randomness of pension plans we should consider is the future lifetime of an individual. The randomness of the future lifetime T_x of an individual aged x can be modelled in the simplest case by the exponential law of mortality (Cipra 2010):

$${}_t p_x = \exp\{-\int_x^{x+t} \lambda_x ds\} = \exp\{-\lambda_x t\}, \tag{3}$$

where λ_x is the force of mortality at age x (that is, an infinitesimal version of the probability of death at the given age). Life expectancy at age x is:

$$e_x = E(T_x) = \frac{1}{\lambda_x}. \tag{4}$$

A combination of models (2) and (3) produces the present value PV_x of the standard pension (where the unit of pay is an annual payment in continuous time) as random variable:

---

[1] The Value at Risk measure has been introduced to insurance only recently. It is therefore based on data for only one year. While in banking there is access to high frequency data, which allows daily risk measures to be calculated, Value at Risk is calculated by insurers for the whole year and is an assessment of solvency.

$$PV_x = \int_0^{T_x} \exp\{-(\mu t + \sigma B_t)\}\, dt. \tag{5}$$

The probability of ruin, or the probability of an unsustainable pension (Dufresne 1990, Milevsky 1997, 2006), is defined as (Cipra 2010):

$$P(PV_x > w) = P\left(\int_0^{T_x} \exp\{-(\mu t + \sigma B_t)\}\, dt > w\right), \tag{6}$$

where $w > 0$ is the sum in the participant's account at retirement age x, which can be approximated as:

$$P(PV_x > w) \sim \frac{1}{\beta^{\alpha}\Gamma(\alpha)} \int_0^{1/w} z^{\alpha-1} \exp\{-(\tfrac{z}{\beta})\}\, dz = \frac{1}{\beta^{\alpha}\Gamma(\alpha)} \int_0^{w} y^{-(\alpha-1)} \exp\{-(\tfrac{1}{\beta y})\}\, dy, \tag{7}$$

where:

$\alpha = \frac{2\mu + 4\lambda_x}{\sigma^2 + \lambda_x}$, $\beta = \frac{\sigma^2 + \lambda_x}{2}$ and $\Gamma(\alpha)$ is the gamma function,

$\Gamma(\alpha) = \int_0^{\infty} z^{\alpha-1} e^{-z}\, dz.$

7. Simulation Analysis

The formulas presented in Section 2 enable us to perform the corresponding calculations for pension plans in Poland. In the simulation study we use:

1. Financial data: the technical interest rate can be used for the purpose of the investment formula (2).
2. Longevity data: we have used life tables for male and female in Poland in 2013. From the expected remaining lifetime (life expectancy) e_x at particular ages x given in these life tables it is easy to estimate the parameters λ_x according to formula (4).
3. Projections of life expectancy at age 65.

Projecting Life Expectancy at Age 65 for Poland

The Lee-Carter method, whose principle is relatively simple, is used to forecast life expectancy. It involves modelling age-specific mortality over time based on the following:

$$\ln(m_{x,t}) = \phi_x + \psi_x \gamma_t + \varepsilon_{x,t}; \quad x = 0, 1, \ldots, k-1; \; t = 1, 2, \ldots, T, \tag{8}$$

where $m_{x,\,t}$ are specific mortality rates at age x and in time t, constituting $k - 1xT$ by dimensional matrix **M** of specific mortality rates at age x and in time t, $e^{\varphi}x$ is the average profile of mortality at age x (irrespective of time t), ψ_x is the age-specific constant that represents the speed of fluctuation of mortality at a given age, as opposed to the total level of mortality γ_t in time t (γ_t can also be described as the total mortality index), and $\varepsilon_{x,\,t}$ is white noise. The identification model is ensured by conditions $\sum_{t=1}^{T} \gamma_t = 0$ and $\sum_{x}^{k-1} \psi_t = 0$. The construction of the forecast is based on the fact that parameters $\hat{\phi}_x$ and $\hat{\psi}_x$ are constant in time and the total mortality index, which is a one-dimensional time series, is modelled and forecast based on the Box-Jenkins methodology (Box & Jenkins 1970). ARIMA models are used to calculate the forecast. Then, using estimates of parameters $\hat{\phi}_x$ and $\hat{\psi}_x$, a forecast of age-specific mortality rates is obtained from the relationship of

$$\hat{m}_{x,t} = \exp\{\hat{\phi}_x + \hat{\psi}_x \hat{\gamma}_t\} \quad x = 65; \;\; t = 2015, 2020, 2025, 2030. \tag{9}$$

The results are presented in Table 3.

Table 3. Projected Life Expectancy at Age 65

e_x	2015	2020	2025	2030
Male	15.75	16.03	16.69	17.25
Female	19.99	20.58	21.43	22.61

Source: authors' own calculations in the R programming language.

The Probability of Ruin

According to formula (6), we have calculated the probability of ruin (the probability of an unsustainable pension) for a retirement age of 65 depending on a spending rate of $1/w$. In this way, a spending rate of 0.06 would mean, for example, that a pension account of PLN 500,000 would pay PLN 30,000 annually and PLN 2,500 monthly. Table 4 and Table 5 present results for a retirement age of 65 only. The calculations are performed separately for males and females, and for various values of investment drifts and volatilities: $\mu = 1\%$ and $\sigma = 5\%$; $\mu = 2.25\%$ and $\sigma = 5\%$; and $\mu = 5\%$ and $\sigma = 10\%$.

Table 4. Probability of Ruin for Male (i.e. Probability of an Unsustainable Pension in %) for Retirement Ages and Spending Rates for Different Strategies

Year of projection	w									
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
Strategy no. 1 $\mu = 1\%$, $\sigma = 5\%$										
2015	0.4	2.6	7.4	14.3	22.7	34.7	41.1	49.9	58	65.2
2020	0.4	3.0	8.4	16.0	25.2	35.9	44.5	53.5	61.7	68.8
2025	0.5	3.1	8.5	16.3	25.6	37.4	45.0	54.1	62.3	69.4
2030	0.4	2.9	8.0	21.7	32.9	44.2	54.7	64.0	71.9	78.4
Strategy no. 2 $\mu = 2.25\%$, $\sigma = 5\%$										
2015	0.1	1.2	4.0	8.8	15.3	23.0	31.4	39.9	48.1	55.9
2020	0.1	1.2	4.2	9.0	15.6	23.4	31.8	40.4	48.7	56.5
2025	0.2	1.3	4.6	9.9	17.1	25.4	34.3	43.4	51.8	59.7
2030	0.4	1.1	6.1	13.0	21.9	32.0	42.2	52.0	61.0	68.7
Strategy no. 3 $\mu = 5\%$, $\sigma = 10\%$										
2015	0	0.4	1.7	4.2	8.0	13.0	19.0	25.6	32.7	39.8
2020	0	0.4	1.7	4.3	8.2	13.3	19.5	26.2	33.3	40.5
2025	0	0.5	2.0	4.8	9.1	14.7	21.4	28.7	36.3	43.8
2030	0	0.4	2.3	5.8	11.0	16.7	25.2	33.3	41.7	49.7

Source: authors' own calculations in the R programming language.

Table 5. Probability of Ruin for Female (i.e. Probability of an Unsustainable Pension in %) for Retirement Ages and Spending Rates for Different Strategies

Year of projection	w									
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
Strategy no. 1 $\mu = 1\%$, $\sigma = 5\%$										
2015	0.5	4.2	11.3	21.0	32.0	43.2	53.6	62.9	70.8	77.4
2020	0.2	4.4	11.7	21.7	32.9	44.2	54.7	64	71.9	78.9
2025	0.6	4.5	12.1	22.4	33.8	45.2	55.8	65.1	72.9	79.3
2030	0.7	4.8	12.8	23.6	35.4	47.1	57.7	66.9	74.6	80.8
Strategy no. 2 $\mu = 2.25\%$, $\sigma = 5\%$										
2015	0.2	1.7	5.9	12.6	21.4	31.2	41.3	51.0	59.9	67.7
2020	0.1	1.9	6.1	13.0	21.9	32.0	42.3	52.0	61.0	68.7
2025	0.2	1.9	6.5	13.9	23.3	33.7	44.3	54.3	63.2	70.9
2030	0.2	2.0	6.6	14.1	23.6	34.1	44.8	54.7	63.7	71.4

Table 5 cnt'd

Year of projection	w									
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
Strategy no. 3 $\mu = 5\%$, $\sigma = 10\%$										
2015	0.0	0.6	2.3	5.6	10.7	17.1	24.6	32.7	40.9	48.0
2020	0.0	0.6	2.3	5.7	10.9	17.5	25.1	33.3	41.7	49.7
2025	0.0	0.6	2.3	5.8	11.0	17.7	25.3	33.6	41.9	50.0
2030	0.0	0.6	2.5	6.1	11.7	18.6	26.6	35.1	43.7	51.9

Source: authors' own calculations in the R programming language.

The results provide the following very interesting conclusions. Under a conservative investment strategy with parameters $\mu = 1\%$ and $\sigma = 5\%$, the probability that a man from Poland with a retirement age of 65 and a spending rate of 0.06 (an annual PLN 30,000 from a pension account of PLN 500,000) will face an unsustainable pension is 34.7% in 2015 and 44.2% in 2030. The probability is higher for a female of the same age: 43.2% in 2015 and 47.1% in 2030. If the investment drift increases, the probability of ruin falls considerably: for $\mu = 5\%$ and $\sigma = 10\%$, for example, the probability of ruin for males is only 13.0% in 2015 and 16.7% in 2030, while that for females is 17.1% in 2015 and 18.6% in 2030. This means that in 2015 only one in ten males and one in five females is ruined before death.

8. Conclusions

Demographic ageing must be understood as presenting a new challenge to society. There are a number of issues to be confronted if it is to cope with double the number of senior citizens, not the least of which is the rearrangement of systems of social and health care. It is important to remember that Poland is gradually becoming a longevity society. Unfortunately, gains in mortality and life expectancy are uncertain. Longevity risk, for example, which is defined as the uncertainty surrounding future developments in mortality and life expectancy, presents the threat that people will outlive the funds available to support them in retirement.

With respect to the randomness of investment activities and longevity, the model presented in this paper makes it possible to investigate the probability of the unsustainability of pensions. It provides numerical confirmation that this probability decreases as the age of retirement increases (Trzpiot & Majewska 2016), that it decreases as the spending ratio decreases (in particular where there is an increasing pension account and there are decreasing annuity payments), that it decreases as investment drift increases, that it decreases as investment volatility decreases, and that it is always lower for males than for females.

Bibliography

Antolin, P. (2007) *Longevity Risk and Private Pensions*. OECD Working Papers on Insurance and Private Pensions No. 3, Paris: OECD, https://www.oecd.org/pensions/insurance/37977228.pdf. Accessed: 5 March 2015.

Arltová, M., Langhamrová, J. and Langhamrová J. (2013) "Development of Life Expectancy in the Czech Republic in the Years 1920–2010 with an Outlook to 2050". *Prague Economic Papers* 22(1): 125–43, https://doi.org/10.18267/j.pep.444.

Barrieu, P., Bensusan, H., Karoui, N. E., Hillairet, C., Loisel, S., Ravanelli, C. and Salhi, Y. (2012) "Understanding, Modelling and Managing Longevity Risk: Key Issues and Main Challenges". *Scandinavian Actuarial Journal* 3: 203–31, https://doi.org/10.1080/03461238.2010.511034.

Box, G. E. P. and Jenkins, G. M. (1970) *Time Series Analysis, Forecasting and Control*. San Francisco: Holden-Day.

CIA (2014) *Factbook*, https://www.cia.gov/library/publications/the-world-factbook/fields/2102.html. Accessed: 5 March 2015.

Cipra, T. (2010) *Financial and Insurance Formulas*. Heidelberg–Dordrecht–London–New York: Physica-Verlag, Springer.

Drever, F., Whitehead, M. and Roden, M. (1996) "Current Patterns and Trends in Male Mortality by Social Class (Based on Occupation)". *Population Trends* 86: 15–20.

Dufresne, D. (1990) "The Distribution of a Perpetuity with Applications to Risk Theory and Pension Funding". *Scandinavian Actuarial Journal* 1: 39–79, https://doi.org/10.1080/03461238.1990.10413872.

Goldman, N. (2001) "Social Inequalities in Health: Disentangling the Underlying Mechanisms" in M. Weinstein, A. Hermalin, and M. Stoto (eds) *Population Health and Aging: Strengthening the Dialogue between Epidemiology and Demography*. New York: New York Academy of Sciences, pp. 118–39.

Graziani, G. (2011) "Live Long and Prosper". *Benefits Canada*, December: 64–65.

IMF (2012) "The Financial Impact of Longevity Risk" in *Global Financial Stability Report*. Washington: World Economic and Financial Surveys, pp. 123–53.

Lee, R. D. and Carter, L. R. (1992) "Modeling and Forecasting U.S. Mortality". *Journal of the American Statistical Association* 87 (14): 659–71.

Malliaris, A. G. and Brock, W. A. (1982) *Stochastic Methods in Economics and Finance*. Amsterdam: North-Holland.

Milevsky, M. A. (1997) "The Present Value of a Stochastic Perpetuity and the Gamma Distribution". *Insurance: Mathematics and Economics* 20 (3): 243–50, https://doi.org/10.1016/s0167-6687(97)00013-9.

Milevsky, M. A. (2006) *The Calculus of Retirement Income*. Cambridge: Cambridge University Press.

NAIC (2010) *Longevity Risk*. National Association of Insurance Commissioners and the Center for Insurance Policy and Research, http://www.naic.org/cipr_topics/topic_longevity_risk.htm. Accessed: 10 March 2015.

Osorio, J. (2013) *Mortality Improvements and Longevity Risk*. Swiss Re, http://www.fasecolda.com/files/5313/8912/6058/05-006-fasecolda_2013_seminario_de_vida.pdf. Accessed: 10 March 2015.

Population Projection 2014–2015 (2014). Statistical Analyses and Studies. Warsaw: Central Statistical Office, www.stat.gov.pl. Accessed: 12 March 2015.

Pension Fund Risk Management: Financial and Actuarial Modeling (2010) eds M. Micocci, G. N. Gregoriou and G. B. Masala. Boca Raton: Chapman and Hall/CRC.

Swiss Re (2014) *A Short Guide to Longer Lives: Longevity Funding Issues and Potential Solutions*, http://bvzl.de/Newsletter/2014_12/Spotlight_02.pdf. Accessed: 10 March 2015.

Trzpiot, G. and Majewska, J. (2016) "Modeling Longevity Risk". *Studia Ekonomiczne. Zeszyty Naukowe Uniwersytetu Ekonomicznego w Katowicach* 288, *Informatyka i Ekonometria* 2016 (5): 33–46.

Wong-Fupuy, C. and Haberman, S. (2004) "Projecting Mortality Trends: Recent Developments in the United Kingdom and the United States". *North American Actuarial Journal* 8 (2): 56–83 https://doi.org/10.1080/109202777.2004.10596137.

Abstract

Modelowanie ryzyka długowieczności w świetle prognozy dla Polski do 2050 roku opracowanej przez Główny Urząd Statystyczny

Starzenie się społeczeństwa jest zjawiskiem, z którym mierzą się wszystkie kraje wysoko rozwinięte. W artykule analizujemy prawdopodobieństwo (zwane prawdopodobieństwem trwałej emerytury) wyczerpania zgromadzonych środków finansowych w okresie emerytalnym.

Słowa kluczowe: ryzyko długowieczności, projekcje przeciętnego trwania życia, program emerytalny o określonej wysokości składek, prawdopodobieństwo ruiny.